The Greeks

Delta, Gamma, Theta, Kappa, Rho

2016

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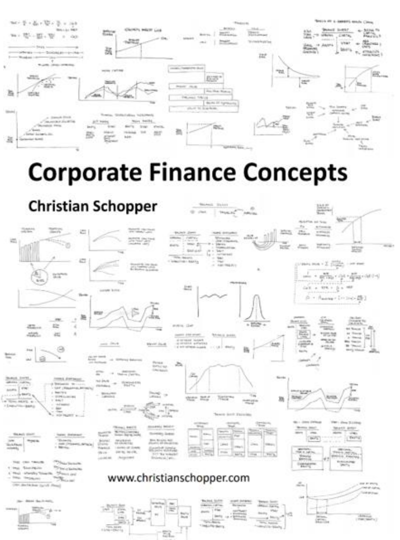

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The Challenge of Measuring Risk and Return

- **Derivatives are a game of risk**
  - **No policy** and **no model** can **eliminate** that element
- It is the one **who controls** best an exposure who **wins**
- There are **many reasons why exposure can go out of control**
  - More **emphasis** is placed **on** features **promoting sales** than on risk control
  - The internal **control system** is **defective** or nonexistent
  - There is **poor top management oversight** of what takes place on the trading floor

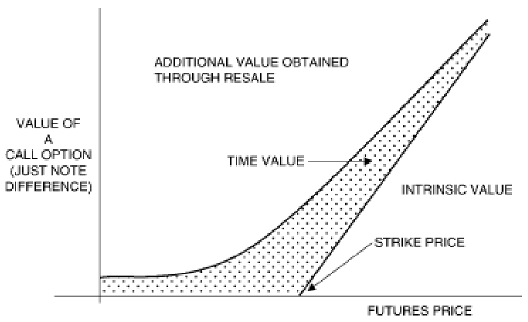
- To simplify matters, many traders (and, sometimes, risk managers) assume that the original **hedge ratio was right** and adjustments are needed only for market gyrations …
- But, this **may not be so** for two reasons:
  - With little price history to show how **volatile** an asset has been in the past, it is more than usually **hard to predict** how volatile it may be in the future
  - Pricing models are based on the false **hypothesis** that the **market behavior** is generally **symmetric and** price variations are **normally distributed**—which is **rarely the case**

Intrinsic Value and Time Value

Intrinsic is the value of the option **if** it were to **expire immediately**

- Essentially, this is the amount the **futures price** is **higher than a call**'s exercise price **or** …
- … **lower than a put**'s exercise price

- In the money
- At-the-money
- Out of the money

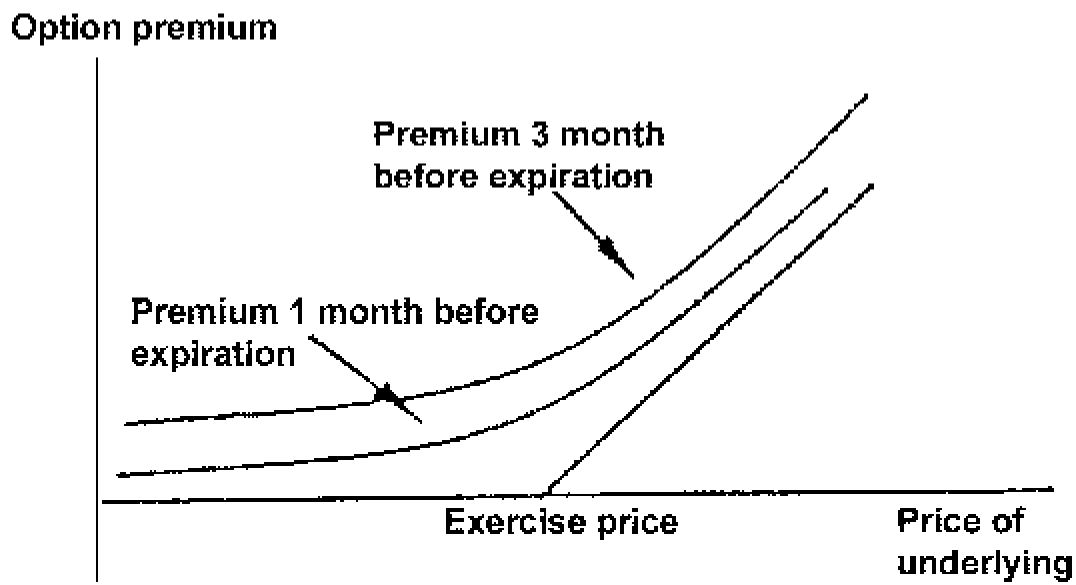


Extrinsic *value* of an option is its **current price less** its **intrinsic value**

- Extrinsic value is also called ***time value*** because the time remaining for the option to make a move is key to its worth

- Time value **is a risk premium** demanded by the option writer, and it depends on
  - The relationship of the **futures price to the exercise price**,
  - The **volatility** of the futures price, and
  - The amount of **time** remaining until expiration

Time Value


Option premium
Premium 3 month
before expiration
Premium 1 month before
expiration
Exercise price
Price of
underlying

The Greeks in a Nutshell

Delta

- Expected change in an **option's price** as a proportion of a **small change in the underlying**
  - Mathematically, it is the first derivative of price change

Gamma

- Partial **derivative of *delta*** and the second derivative of the price function identifying the **speed of change** or the **slope of the curve**

Theta

- Rate at which an option loses computed value for each day that passes with no movement in the price of the underlying, expressing **decay**

Kappa (or *vega*, *lambda*, or *beta prime*)

- Impact of fairly small changes in **volatility**
  - For example, the impact of a 1 percent change in volatility (beta)

Rho (or *phi*)

- The option's **carrying cost**
  - It tells the change in the option price for a 1 percent change in **interest rates**

The Greeks in a Nutshell (cont´d)

Taken together, the five Greeks provide a framework for risk measurement by means of ***sensitivity analysis*** that helps to quantify the risk of an option:

- **Delta** gives the **sensitivity to the asset price**
- **Gamma** gives the **sensitivity of delta to the asset price**
- **Theta** gives the time premium connected to the option's **expiration**
- **Kappa** gives the **sensitivity to volatility**
- **Rho** gives the **sensitivity to interest rates**

The Greeks' Background

- The option's price consists of its *intrinsic value* (if any) …
  - The value of the option if it were exercised immediately
- … and its *time value*
  - The time to the option's maturity
- **The greater the intrinsic value, the more responsive the instrument is to change in the futures' price**
  - As a metric, delta addresses itself to this price dependency

The Greeks in a Nutshell (cont´d)

Option Parameters

Option Parameters			Call Option	Put Option
Underlying Price	100	Theoretical Price	2.649	0.329
Exercise Price	98	Delta	0.798	-0.202
Days Until Expiration	30	Gamma	0.098	0.098
Interest Rates	5	Gamma 1%	0.01	0.01
Dividend Yield	1	Vega	0.081	0.081
Volatility	10	Theta	-0.024	-0.011
Rounding	3	Rho		
Graph Increment	1			

Calculate

http://www.option-price.com/

The Delta

- **Delta Neutral describes a portfolio of related financial securities, …**
- **… in which the portfolio value remains unchanged …**
- **… when small changes occur in the value of the underlying security**

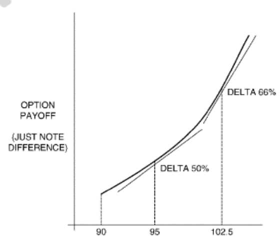
- **A long call position may be delta hedged by shorting the underlying stock**
- **This strategy is based on the change in premium (price of option) caused by a change in the price of the underlying security**

- For instance, an **option with a delta of 0,4 can be expected to change its value at 40% the rate of change in the price of the underlying security**
  - If the underlying security goes up 5, the option's theoretical value can be expected to go up 2

- With **higher volatility**, the **delta** is **somewhat higher**, and with a lower volatility, somewhat lower

- One general **observation** is that **if** the **underlying price** is **about the same as the strike** price, the **options premiums will vary by about 50% the change in the underlying** contract

- Delta is also known as the ***hedge ratio*** because it expresses the ratio of the underlying to the option contract, for reasons of neutral hedge ("**Delta Hedging**")
  - Delta-neutral, gamma-neutral, and other positions in relation to the aforementioned metrics are established through hedging, but not all of the above metrics can be hedged at the same time

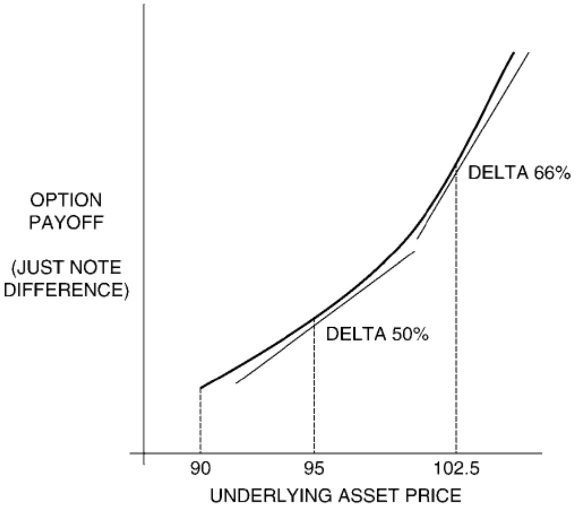


Delta Hedging

Delta is the measure of percentage change in the price of an option for a unit change in the price of the underlying

$$\frac{dF(x)}{dx}$$

- The value of delta ranges from 0 to 1
  - A value of 0 would result from a **far out-of-the-money option** for which there is **no need to hold a hedge** in the underlying asset since the probability of exercise is virtually nil
  - In contrast, a value of 1 would come from a **deep in-the-money option** that is virtually certain to be exercised…
  - … therefore, the option writer would have to **hold the underlying as a hedge** against the option he or she had sold



- If a trader **buys a call** with the delta of 0.25, in **theory** he or she is **long** 0.25 of an **underlying futures** contract
- The delta identifies the theoretical or equivalent futures position and therefore the change in the theoretical value of an option with respect to the change in the price of the underlying contract

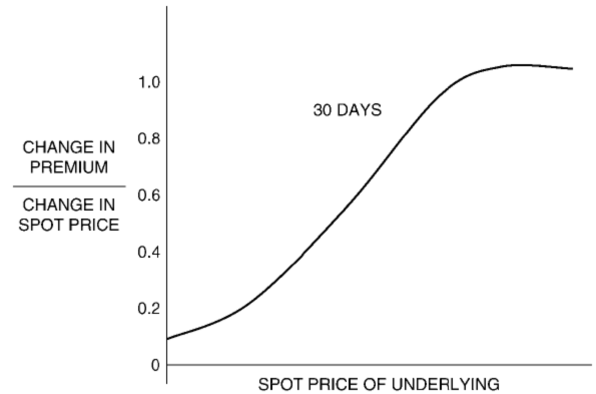
Delta Hedging (cont´d)

- Options **at-the-money** usually have a delta of **0.5**
  - … which means that for a price change of 1 in the underlying instrument, the option price moves by 0.5
  - In an at-the-money call where the spot price is close to the strike price, the delta will rise with the spot price
  - The delta increases more rapidly as the expiration date approaches
- Options further **in-the-money** have deltas **greater than 0.5**
  - … converging on 1.0 as time to expiry approaches
  - For a deep-in-the-money call where the spot price is far above the strike price, the delta is 1
- In contrast, options **out-of-the-money** tend to have deltas of **less than 0.5**
  - … and they converge on 0.0 as expiry draws closer

Delta Hedging (cont´d)

As the underlying asset moves, the delta also changes

- In the case of a call, a rise in the underlying asset - that increases the probability of exercising – makes delta rising
- Because delta gives the price change in the option for a 1% change in the underlying asset, the delta value of a position is **used to estimate the value at risk for small changes in prices**
- The fact is that as a metric, delta is fairly well understood by market players



- Experts contend that a **static delta hedge can be unreliable**, especially in volatile markets
  – For instance, the delta of a hedge might drift with **movements** in the spot exchange rate because of interest rate changes, balance-of-payments deterioration, or political events
  – **This is** one of the reasons **why delta hedges** are **implemented with** instruments such as **forward contracts** and **futures contracts** that are not sensitive to changes because of movements in the spot rate

Excursion – Delta Hedging: How to Delta Hedge with Options

The simplest delta hedge just involves trading the underlying …

- The **underlying** has a **delta of 1** (or 100%)
- **If** you are **long the underlying** (maybe some shares or an ETF or some commodity futures), your **delta is simply the amount of the underlying** you are long
- And the **simplest** possible **delta hedge**, would be to **<u>sell</u>** out some or all of your longs

… but you can also delta hedge using options

- The biggest advantage of hedging with options is that you can create **more interesting hedge results** that are tailored to the portfolio you want to create

How do we delta hedge with options?

- **Calls** have a **positive delta** and **puts** have a **negative delta**
- **If** the **underlying** product **rallies** in price, **calls** become **more valuable** and **puts** become **less valuable**
- Now in order to **delta *hedge*** with options, we need to **trade** so that the **P&L from our options** - when the underlying moves -, **is the *opposite* to the P&L from our underlying portfolio**
- That is the nature of a hedge!
- So, **if** we are **long the underlying**, an option **delta hedge** will involve either **selling calls or buying puts** or a combination of both
- If the underlying price falls, our short calls/long puts will *make money* back for us

Excursion – Delta Hedging: Why use Options to Delta Hedge

Example

- We have a portfolio which is **long some shares** on a company
- We think the shares are a **good investment medium term**, but we are concerned about the forthcoming earnings call
- There are **rumours that the results for the previous quarter were disastrous**

1. Generally, **we want to stay long** the shares
2. We think there is a **chance the shares will fall** after the earnings call
3. We think there is a **slim chance that the shares will plummet** if the earnings are dreadful **and** that there is a price below which **we really would rather not own the shares anymore**

Next Steps

What shall / can we do?

Excursion – Delta Hedging: Why use Options to Delta Hedge

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Next Steps

- Hedging this risk just by trading the underlying is not really possible
  - You either sell some or all of the shares
- Instead, use options to create a more tailored hedge strategy:
- For example, we could just **buy some out-of-the-money put options**
  - Let's say we buy some puts with a **10% delta** that expire in a **few weeks**
  - **If the stock rallies**, we will make the same profit on the shares, but probably lose some (or all) of the money we spent on the hedging puts, that's the cost of the hedge
  - **If the stock dips** just a little, then our puts *should* increase in value and cover some of the *losses* on our long shares
  - And **if the stock** really **falls out of bed**, our losses are capped once we hit the put strike
  - **If the stock falls below the put strike**, we can simply exercise our puts and therefore sell the underlying at the strike price

Excursion – Delta Hedging: Some Risks to think about

- The main risk is due to the fact that the **value of options *before expiration*** is **affected by several factors** - and **<u>not</u> just the price of the underlying …**
- For example, **implied volatility**
  - It is possible for the **underlying to fall** in price and for the value of **puts to *fall*** rather than increase …
- Another factor is **time**
  - **Time** affects option values and as it **passes**, their **value** can **fall**
- It is true however that **option values at expiration are all about the price of the underlying**
  - So, if you intend to hold your option delta hedge until the options expire, you can simply use the **strike** of the options **as the pivot point**
  - **At expiration, options are either worth nothing** if they are out-of-the-money **or the difference between their strike and the underlying price** if they are in-the-money

- From this perspective, delta hedging with options is very simple …
- … it is only in **the days and weeks before expiration** that things are slightly more **complicated**
- **Framework** for a successful delta hedge:
  - What does the delta neutral aspect of delta neutral hedging mean?
  - How many shares of the underlying should one short?
  - How many options should be bought?
  - How far out should I purchase the options expirey for?
  - What strike price should I choose for the options?
  - How often should I re-balance the portfolio to make it delta neutral?
- These complex questions require **experience and vision** …
- … not just mathematical analysis and simplification …

Delta Hedging (cont´d)

- For example, a precious metals dealer might **sell a call** option on gold, resulting in a negative gold delta
- To mitigate this exposure, he then **purchases** enough gold **futures** to offset the short option's negative delta
- **Together**, the short option and long futures have a combined gold **delta of zero**
- The slope of graphs is the exact the opposite of each other, so the two positions have equal but opposite deltas
- The market value of the combined position is indicated in the bottom graph. A **tangent line has zero slope**, indicating the position is delta hedged

Delta - Gamma Hedging

- **But**: In the third graph the exposure to the price of gold has not been entirely eliminated

- While the position's delta is hedged, it **still has negative gamma**, **and likely negative kappa** (vega) as well

- Such residual gamma and kappa (vega) exposures are inevitable when options positions are delta hedged

- One solution is **Delta - Gamma Hedging**, in which options are added to a portfolio to **achieve both a zero delta <u>and</u> zero gamma**
  - Not only will this eliminate gamma exposure, but it will largely address vega exposure as well
  - **Because options can be expensive, dealers rarely employ delta-gamma hedging**

Gamma Hedging

- Bond convexity is a measure of the sensitivity of the duration of a bond to changes in interest rates, …
- … the second derivative of the price of the bond with respect to interest rates

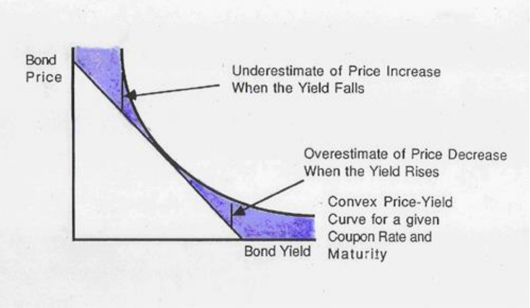
- Gamma, also known as the **option's curvature**, expresses the **change in** the **delta** of the option induced by a small change in the price of the underlying asset

$$\frac{d^2F(x)}{dx^2}$$

- Hence, Gamma measures **delta's sensitivity to changes**, to the underlying contract's value
  - Therefore, the maintenance of a delta-neutral position requires important adjustments given that delta is sensitive to the currency's price

- ***If* an option has a delta of 0,75 and a gamma of 0,10, *then* the option's expected delta will be 0,85 if the underlier goes up 100 bps**
  - It will be 0,65 if the underlier goes down 100 bps

- Gamma for options is **analogous to convexity for bonds**
  - While the target **option** may have a positive **gamma**, currency **forward** and **futures** contracts have gamma equal to **zero**
  - Among currency options, those with the shortest remaining time to expiration have the largest gamma

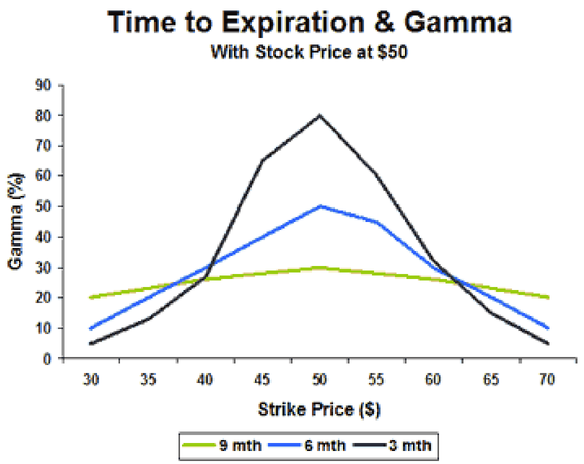


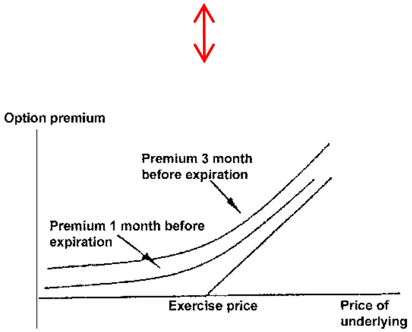
- The risk of delta changing is referred to as the *gamma risk*

Gamma Hedging (cont´d)

Passage of time and its effects on the gamma

- As the time to expiration draws nearer, the gamma of at-the-money options increases while the gamma of in-the-money and out-of-the-money options decreases.

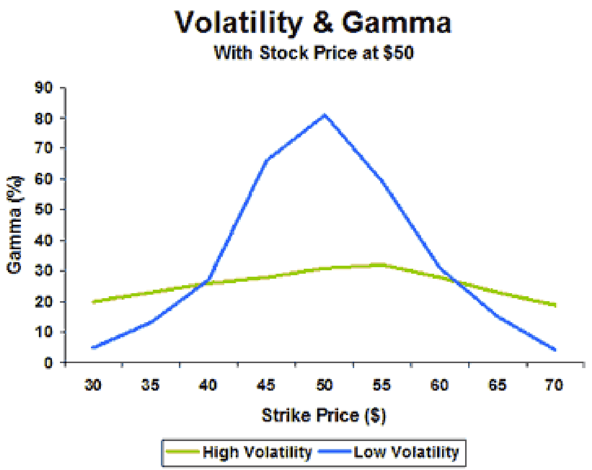




Gamma Hedging (cont´d)

Changes in volatility and its effects on the gamma

- When **volatility** is **low**, the **gamma** of at-the-money options is **high** while the gamma for deeply into or out-of-the-money options approaches 0
  - Because when volatility is low, the time value of such options are low but it goes up dramatically as the underlying stock price approaches the strike price.
- When **volatility** is **high**, **gamma** tends to be **stable across all strike prices**
  - This is **due to** the fact that when volatility is high, the **time value** of deeply in/out-of-the-money options are already quite **substantial**
  - Thus, the increase in the time value of these options as they go nearer the money will be less dramatic and hence the low and stable gamma



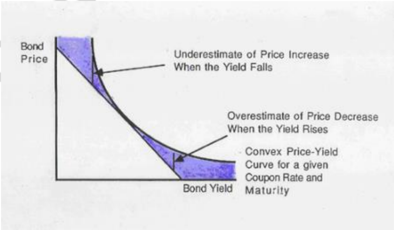
Gamma Hedging (cont´d)

- **Delta hedging provides protection against relative small changes in the price of underlying b/w (better / worse) rebalancing. …**
- **… whilst Gamma neutrality provides protection against larger movement in this stock price**

- **Higher Gammas mean greater potential loss for sellers and, for buyers, greater potential gain …**
- **… particular in case of large stock price movements**

- **A *positive gamma* is a buy signal for calls and puts**
  - The significance of a positive gamma is that the **delta** of the position **varies in the same sense as** that of the **underlying**
  - It increases when the price of the underlying increases, and drops when the payoff function drops
- **A *negative gamma*, in contrast, is a sell signal for calls and puts**
  - In order to remain delta neutral, the trader is obliged to buy the underlying if its price rises and sell if its price drops

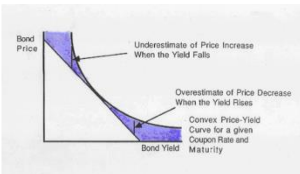
- In the case of both positive and negative **gamma**, its **importance** is **greater when** its **absolute value** is **greater—because it measures the acceleration (or deceleration) of the option**
  - How fast the option picks up or loses speed (hence delta) as the price of the underlying contract rises or falls



Three **approaches** are used to *hedge gamma*

- **Buy back options** identical to the ones that have been sold
  - Such **back-to-back deals** are not creating any profits, and therefore they are very **rare** in the OTC market
- **Buy deep-out-of-the-money options**
  - A practice known as *buying the tails*. This applies to portfolios with at-the-money or slightly out-of-the-money options
- Do a **horizontal spread**

Gamma Hedging (cont´d)



Deep Out Of The Money Options

Example: If the current price of the underlying stock is $10, a put option with a strike price of $5 would be considered deep out of the money

- While a deep of out the money option seems worthless, it still holds some value
  - All options, both in and out of the money, contain **time value**
- Time value measures the benefit of having an option with time remaining until maturity
  - So, while a deep out of the money call or put has no intrinsic value, some investors may be willing to pay a small amount for the remaining time value
  - However, this time value decreases as the option moves closer to its expiry date

Horizontal Spread

Example: **Purchase** of a Dec 20 **call** and the **sale** of a June 20 **call**

- An options strategy involving the simultaneous purchase and sale of two options of the **same type**, having the **same strike price**, but **different expiration dates**
- This strategy is used to **profit from** a **change in the price difference as the securities move closer to maturity**
  - The horizontal spread can be used to attempt to take advantage of a **difference in the implied volatilities between two different months' options**
  - Trader will ordinarily implement this **strategy when the options bought have distinctly lower implied volatility than the options written (sold)**

Delta – Gamma Hedging, again

- *Delta-Gamma hedge* uses **options near to expiration for convexity** …
- … taking a **position in forward and futures contracts to match the delta of the target option**

- The effects of changes in the spot exchange rate on the option's delta are captured by gamma, which critically depends on the time remaining until expiration
  – Close-to-expiration options can be easily found in the market, and therefore be nicely incorporated

- Example: The trader may take a position in a short-lived call to match the gamma of the target longer-life option
  – Given that short-lived calls have much larger gamma than long-lived calls, few of them will be required

- The good news is that delta-neutral and gamma-neutral positions help in dynamic hedging
  – Essentially, while dynamic hedging is a protective strategy, it can have the effect of transferring risk by amplifying it

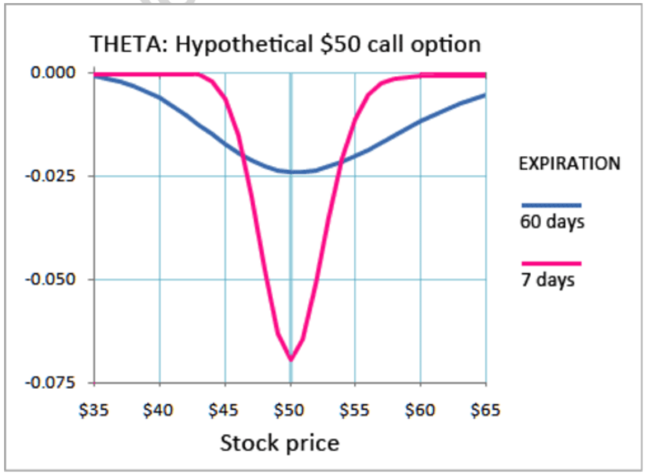
- It helps as the market hits the players, particularly those who want to delta hedge in the same direction at the same time because
  – contrarians disappear from the market, and
  – there are no takers on the other side of the trading

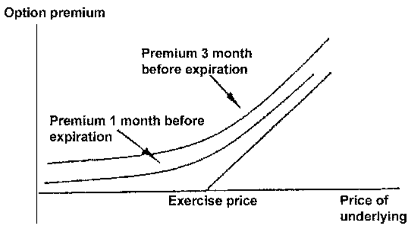
Theta

- It is a fact of life that all options, no matter what strikes or what markets, will always have zero time value at expiration
- *Theta* will have wiped out all extrinsic value leaving the option with no value or some degree of intrinsic value

Theta

- It quantifies the **loss** of the option's computed **value for each day with no movement in the price of the underlier**
  - Theta exposure is closely related to gamma exposure.
- The theta factor is sometimes referred to as the option's ***time decay*** …
- … but in reality, it **reflects upon price stability, rather than turbulence**
- Theta is **always negative**
  - Therefore, it benefits the writer and erodes the value held by the buyer of an option
  - It **becomes zero at expiration** of the option, **decaying most rapidly toward the end of an option's life**





Theta Hedging

- The rate of **decay decreases in** the **more distant contract months**
  - The yellow highlights the calls that are at the money and the violet the at-the-money puts
  - The January 110 calls, for example, have a *Theta* value of -7.58, meaning this option is losing $7.58 in time value each day

Options	JAN <21>			FEB <49>			APR <112>			JUL <203>		
130.0 calls	0.04	-0.15	0.04	0.13	-0.57	0.13	0.88	-1.47	0.88	2.22	-1.60	2.22
125.0 calls	0.08	-0.81	0.08	0.35	-1.57	0.35	1.63	-2.08	1.63	3.30	-1.92	3.30
120.0 calls	0.30	-2.85	0.30	0.98	-2.89	0.98	2.82	-2.69	2.82	4.91	-2.25	4.91
115.0 calls	1.10	-5.76	1.10	2.24	-4.19	2.24	4.61	-3.16	4.61	6.84	-2.46	6.84
110.0 call>	3.10	-7.58	3.01	4.41	-4.90	4.32	7.00	-3.40	6.91	9.26	-2.57	9.17
105.0 calls	6.36	-6.34	1.27	7.67	-4.92	2.58	10.00	-3.39	4.91	12.26	-2.60	7.17
100.0 calls	10.58	-4.19	0.49	11.56	-4.29	1.47	13.55	-3.17	3.46	15.56	-2.50	5.47
95.0 calls	15.30	-2.48	0.21	15.97	-2.89	0.88	17.56	-2.86	2.47	19.27	-2.35	4.18
90.0 calls	20.29	-1.50	0.20	20.60	-2.06	0.51	21.86	-2.27	1.77	23.27	-2.15	3.18
130.0 puts	20.20	0.00	0.29	20.05	0.00	0.14	20.40	-0.43	0.49	21.20	-0.74	1.29
125.0 puts	15.07	0.00	0.16	15.20	-0.41	0.29	15.94	-1.18	1.03	16.98	-0.90	2.07
120.0 puts	10.19	-1.69	0.28	10.77	-1.85	0.86	11.98	-1.67	2.07	13.47	-1.21	3.56
115.0 puts	5.95	-5.28	1.04	7.04	-3.19	2.13	8.74	-2.16	3.83	10.40	-1.45	5.49
110.0 puts>	2.90	-6.86	2.90	4.24	-3.95	4.24	6.10	-2.41	6.10	7.84	-1.59	7.84
105.0 puts	1.20	-5.82	1.20	2.32	-3.77	2.32	4.10	-2.41	4.10	5.78	-1.62	5.78
100.0 puts	0.43	-3.52	0.43	1.22	-3.06	1.22	2.72	-2.23	2.72	4.19	-1.57	4.19
95.0 puts	0.15	-1.82	0.15	0.60	-2.13	0.60	1.75	-1.90	1.75	3.03	-1.45	3.03

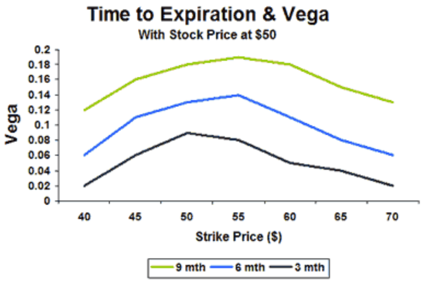
- If a trader desires less time premium risk and a back month option is chosen, the trade-off is that more premium is at risk from *Delta* and *Kappa* risk
  - You can slow down the rate of decay by choosing an options contract with more time on it, but you add more risk in exchange due to the higher price (subject to more loss from a wrong-way price move) and from an adverse change in implied volatility (since higher premium is associated with higher *Vega* risk)

Kappa (*Vega*, *Lambda*, or *Beta prime)*

Kappa

- Expresses the sensitivity of an option's computed value to **small changes in volatility** affecting a given position
- An option with a kappa of 0.20 can be expected to gain (lose) 0.20 in theoretical value for each percentage point increase (decrease) in volatility
- *If* kappa is **high**, ***then*** an option's value is **sensitive to small changes in volatility**
- *If* kappa is low, *then* changes in volatility have little impact on the option's value

- Kappa values range **between zero and infinity**, **declining as** the option's **expiration** approaches
  - Such values impact on the option's premium
  - **Longer dated options** have a **higher kappa** because they are **more sensitive to changes in *implied volatility***
  - Kappa-neutral positions, if they ever exist, are supposed to make investors indifferent to shifts in volatility parameters

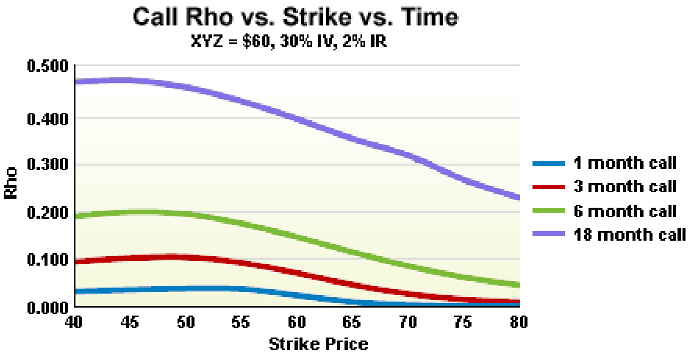


Rho *(Phi)*

Rho

- Reflects the option's **carrying cost**
  - It gives the change in the option price and premium per 1 percent change in *interest rate*

- Some traders think that since interest rates are relatively unimportant in the evaluation of options on futures, but this is not necessarily true
  - The **relation** between stock option premiums and interest rates is **positive**
  - In contrast, the relation between options on futures and interest rates is negative



- With currency options, for example, there are two separate interest rates to be measured—one of the base currency; the other of the quoted currency—because the forward exchange rate depends on the ratio between the two interest rates

- The intrinsic value component will change, and it is important to know about the direction and magnitude of impact

Summary

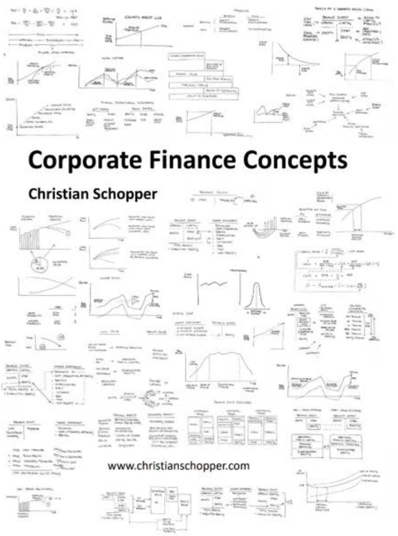
Several variables influence option prices:

- **Amount of volatility**
  - Increase in volatility usually is positive for put and call options, if you're long in the option
- **Changes in the time to expiration**
  - With expiration, the more negative the time factor becomes for a holder of the option and the less your potential for profit
- **Changes in the price of the underlying asset**
  - An increase in the price of the underlying asset usually is a positive influence on the price of a call option
- **Interest rates**
  - Interest rates are less important than the other factors most of the time
  - Higher interest rates make call options more expensive and put options less expensive, in general

- ***Delta*** measures the effect of a change in the price of the underlying asset on the option's premium
- ***Gamma*** measures the rate of change of delta in relation to the change in the price of the underlying asset, and it enables you to predict how much you're going to make or lose based on the movement of the underlying position
- ***Theta*** measures the rate of decline of the *time premium* (the effect on the option's price of the time remaining until option expiration) with the passage of time
- ***Kappa (Vega)*** measures risk exposure to changes in implied volatility and tells traders how much an option's price will rise or fall as the volatility of the option varies

Contact

For more concepts click on:



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